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Short Solenoid Lens Focusing Channel for PD Front End

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1. Introduction

Because the Proton Driver, which is under discussion at FNAL for several years, is a high current machine, significant attention is devoted to reducing beam loss in the accelerating channel. Reduction of beam loss means making beam halo as small as it is possible. Because focusing using short solenoids promises lower beam emittance due to its intrinsic azimuthal symmetry, a decision was made to investigate this option for use in the front end part of the proton driver, where beam energy is relatively small and beam is not rigid enough to safely switch to focusing quadrupoles.

The goal of this note is to investigate feasibility of the solenoidal focusing and provide some scaling relations as a base for further development of the focusing channel.

2. Input parameters:

According to the latest linac-based PD specification, warm Drift Tube Linac (DTL) receives beam from RFQ section (Ti = 3.0 MeV) and brings it to a cold single-spoke section (To = 15 MeV). Beam parameters at the input of the 325 MHz DTL were taken similar (but with some reserve) to the measured beam parameters of the existing linac [1]:

- Particle (H-) energy – Ti = 3 MeV
- Average current though macro-pulse – Iav = 25 mA
- Transverse normallized rms emittance ε = 0.3 π·mm·mrad

For the longitudinal parameters, it is assumed:

- Full longitudinal dimension Δφ = 40°
- Full energy spread - ΔE = 40 keV

Relativistic factors used in the note can be found if one knows kinetic energy T or it's equivalent U expressed in eV:

$\gamma = 1 + \frac{T}{E0}$, where $E0 = mc^2$ is the rest energy.

$$\beta = \sqrt{2 \cdot \frac{T}{E0}}$$

3. Focusing solenoid channel concept:

The goal of this section is to understand feasibility of the channel and make a first estimate of the solenoids' focusing strength.

Using short solenoids for beam focusing is quite common in electron optical devices, so theory of these devices is well developed, but unfortunately not readily accessible. Lack of correct treatment of this problem was reported, for example in [2]. Martin E. Schulze studied the issue theoretically and using beam tracking codes back in 1984 for the polarized injector of the Bates Linear Accelerator and came to the conclusion that the right way to define effective length is through using the integral of $B(z)^2 \cdot dz$. Earlier studies also point in this direction; for example, in [3] the expression for the focusing length of a short solenoidal lens can be found in the form:

$$\frac{1}{f} = \frac{q}{8m_0U} \cdot \int_{-\infty}^{+\infty} B_0^2(z)dz$$

that also supports what the statement in [2].

Simple considerations that give similar result can be found below.

Focusing effect of a coil is based mainly on its fringe fields. Simple (but not strict though) considerations help to derive an expression for coil focusing length. Fig.1 will help to visualize details of particle motion within a beam.

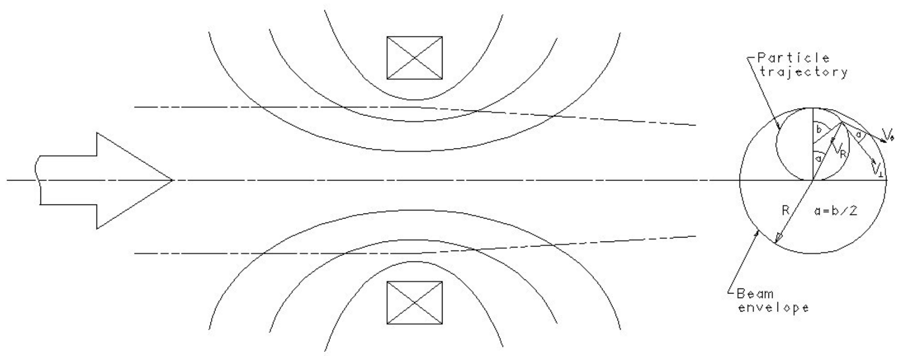

Fig. 1: Beam envelope and particle trajectory projection

Let's consider a particle entering a region with solenoidal field at the distance R from the axis. Let's also consider that the axial field in the center of the region and the radial return field at the edges are separated in space (technically this can be made by adding a cylindrical, ferromagnetic flux return with flat walls at the ends of the region). The particle will see a deflecting component of magnetic field while entering the fringe area. Total transverse pulse due to this field will be:

$$p_\perp = \int_{-\infty}^{0} F \frac{dz}{v_\parallel} = q \cdot \int_{-\infty}^{0} B_\perp(R)dz$$

On the other hand, comparing magnetic flux associated with the longitudinal and transverse field, we obtain

$$\int_{-\infty}^{0} B_\perp(R)dz = \frac{R}{2} \cdot B_c ,$$

where B_c is the field in the center of the lens. So, when the particle comes into the region of pure longitudinal field, its transverse pulse

$$p_\perp = mv_\perp = q \cdot \frac{R}{2} \cdot B_c$$

If magnetic field B_c is much higher then Brillouen equilibrium field, particle trajectory inside the lens is a circle with radius of

$$r = \frac{m}{q} \cdot \frac{v_\perp}{B_c} = \frac{R}{2}$$

While the particle is inside the region of the longitudinal field, this circular movement leads to the rise of radial velocity component v_R. Azimuthal position of a particle is defined by $v_\perp$ and the time particle spends moving through the region with the axial magnetic field. Cross-sectional view in Fig. 1 can be used as a geometry reference for calculating v_R:

$$v_R(z) = v_\perp(z) \cdot \frac{q}{2m\beta \cdot c} \cdot \int_{-\infty}^{z} B_z dl$$

After the particle exits the lens (and if the lens is short enough, which means also that particle's radial position does not change significantly), azimuthal component of particle velocity almost vanishes because it gains the transverse pulse almost opposite to received while entering the field. The radial component does not change because it does not interact with the radial field – this gives the resulting focusing effect. The focusing length can be calculated as

$$f = R \cdot \frac{\beta \cdot c}{v_R} = \frac{4m^2\beta^2c^2}{q^2} \cdot \frac{1}{B_c \int_{-\infty}^{+\infty} B_z dl} = \frac{8 \cdot \frac{m}{q} \cdot U}{B_c \int_{-\infty}^{+\infty} B_z dl}$$

Here U is the energy of particles expressed in eV: $qU = T$.

This expression is not exactly what was obtained in [3] because significant simplifications were made to surface physics that underlies the process of focusing. To derive the expression without significant simplifications we must write down a differential equation of particle motion inside the magnetic field with cylindrical symmetry. For this purpose, radial magnetic field must be expressed in terms of derivatives of the longitudinal field:

$$B_\perp(z,r) \approx -\frac{1}{2} B_0^{'}(z) \cdot r$$

After some transformations, which are out of the scope of this note, the resultant differential equation is

$$\frac{d^2r}{dz^2} = -\frac{q}{8mU} B_z^2(z) \cdot r$$

That leads directly to the expression for the focusing lens from /3/.

So, strength of a short lens is proportional to $\int B_c^{\,2} \cdot dl$ and depends on particle energy. The higher energy, the longer focusing length is. For low energy particles, there can be a problem finding a design solution to the transport problem. For the set of the input beam parameters at the beginning of this note with $\int B_c^{\,2} \cdot dl = 1.0$ T^{2}m , $f \approx 0.5$ m.

The distance between lenses in the transport channel can not be longer than $4 \cdot f$ to insure stable transport condition. In our case, the distance between lenses in the focusing channel must be shorter than ~ 2 m. Longer focusing length means thicker beam at the location of the lenses; to make beam diameter smaller (e.g. to reduce beam loss) will require stronger lenses. At some point the lens' strength will be limited by issues of available s/c wire material or spatial limitations, so careful analysis is required to define requirement for lenses, including some solenoid R&D and particle tracking studies.

If the beam current is significant, space charge adds some defocusing and stronger lenses must be used and less space between the lenses is allowed.

There are some limitations to lens parameter choice.

The first one is the center magnetic field strength. It must be much higher that the Brillouen equilibrium field, which for the beam input parameters is about 1 T. This requirement can be met by using superconducting coils. Another limitation restricts the length of lenses. The lens can provide distortion-free focusing only if it is short. An estimate of a maximal length can be done considering that a particle is allowed to make only 1/8 of a full oscillation inside the lens. Then the time particle spends inside a lens

$$\tau = \frac{\pi \cdot m}{4qB_c}$$

and corresponding length of the lens is

$$L_{\max} = \beta \cdot c \cdot \tau = \frac{\pi}{2\sqrt{2}}\sqrt{\frac{m}{q}} \cdot \frac{\sqrt{U}}{B_c} \approx 1 \cdot 10^{-4} \cdot \frac{\sqrt{U}}{B_c},$$

where U is in eV and B_c is in Tesla. With U = 3 MeV and B_c = 5 T, $L_{max} \approx 3.5$ cm. The length $L_{\max}$ must be considered as the average length:

$$L_{\max} = \frac{1}{B_c} \cdot \int B(z)dl$$

Reducing the central field for several solenoids in the beginning of the channel (where the energy is still low) will allow some increase of their lengths, which probably is good idea taking into the mind the latest result. This will help reducing aberration of the lenses.

The situation becomes better when energy increases. Taking 5 T as the goal field in the center of the solenoid and 0,04 m of the solenoid length, we have lens strength of about 1 T^2m, that is consistent with what we assumed earlier.

So, at this point, there is no obvious show stoppers that would prevent one from building a focusing channel for the from end of the PD based on focusing solenoids. Nevertheless, more studies are required to form the channel (that means to choose right focusing strength, geometrical features, and location of all solenoids in the channel).

4. General Envelope Equation

In this section, an attempt will be made to find a solution for a beam transport channel. The simplest way of doing this I found was by using a general envelope equation [4]. The equation obtained by authors from LLNL is quite general for long beams without taking into the account longitudinal motion.

Main restrictions are:

1. Paraxial approximation
2. Azimuthal symmetry
3. No mass spread in radial direction
4. Magnetic field is uniform within beam profile
5. Only small-angle multiple scattering can be considered.

Previous experience of using the equation showed that it can model behavior of a high current beam quite well, and it is very convenient to use it for making initial estimate of a transport channel. To make final design, more sophisticated means are needed, starting with matrix-type codes (TRACE) and ending by using PIC codes.

If to neglect emittance growth terms (scattering), the envelope equation can be written in terms of mean root-square beam radius:

$$\ddot{R}+\frac{\dot{\gamma}}{\gamma}\cdot\dot{R}+\frac{U}{R}+\frac{\omega_c^2\cdot R}{4}-\frac{C^2}{\gamma^2\cdot R^3}=0$$

where $\omega_c=\dfrac{qB}{m\gamma}$ and for beam in vacuum

$$U=-\frac{qc\mu_0 I_b}{4\pi\beta\gamma^3 m}.$$

Constant C is defined by beam emittance and canonical angular momentum of the beam.

$$C^2 = E^2(t_0) + P^2_\Theta$$

Another constant:

$$P_\Theta=\gamma L+\gamma\omega_c\frac{R^2}{2}=const$$

is canonical angular momentum of the beam, and is defined by a magnetic field on a cathode where L=0.
Emittance E is defined as

$$E^2=\gamma^2R^2\cdot\left[V^2-\left(\dot{R}\right)^2-\left(\frac{L}{R}\right)^2\right]$$

and is also a constant of motion.

It is necessary to mention that definition of emittance here differs from what is widely accepted. It is related to full area the beam occupies in the space mrs radius – mrs transverse velocity. It is normalized, which is reflected by a multiplier γ^2. Corresponding emittance measurement unit is **m²/s**.

For solving application problems, it is better to use longitudinal coordinate as a variable: $z(t) = \int\beta(t)c\cdot dt$. Then we have:

$$R''=-\frac{\gamma'}{\gamma}R'-\frac{q^2}{4m^2c^2}\cdot\frac{B^2}{\beta^2\gamma^2}\cdot R+\frac{q\mu_0}{4\pi mc}\cdot\frac{1}{\beta^3\gamma^3}\cdot I_b\cdot\frac{1}{R}+\frac{E^2}{c^2}\cdot\frac{1}{\beta^2\cdot\gamma^2}\cdot\frac{1}{R^3}$$

If to use more conventional units of m-rad for emittance definition, we can write down finally:

$$R''=-\frac{\gamma'}{\gamma}R'-\frac{q^2}{4m^2c^2}\cdot\frac{B^2}{\beta^2\gamma^2}\cdot R+\frac{q\mu_0}{4\pi mc}\cdot\frac{1}{\beta^3\gamma^3}\cdot I_b\cdot\frac{1}{R}+\frac{\varepsilon_N^2}{\beta^2\cdot\gamma^2}\cdot\frac{1}{R^3}$$

Here ε_N is **rms normallized emittance** measured in "m·rad". Nevertheless, it is still related to the whole area of the beam in phase space (so, no π multiplier exists in the unit string). It is also a motion constant. To switch to effective emittance definition, it will be necessary to use an equation for ellipse area: A = π·a·b.

Now we can define all parameters that depend on z (beam energy and magnetic field, take bean input parameters: R_0, I_b and ε, and solve the envelope equation using an appropriate solver. For the purpose of this work, the energy gain was distributed evenly along the length of the system with average gradient of 2 mV/m. Current value was taken equal to expected maximal current in each ellipsoidal bunch, which is about

$$I_{max} = I_{av} \cdot 4/3 \cdot 360/40 = 300 \text{ mA}.$$

RMS emittance expressed in m·rad:

$$\varepsilon = 1.0 \cdot 10^{-6} \text{ m-rad}.$$

Initial radius of the beam is assumed 5.5 mm that corresponds to rms value of about 2.5 mm.

The equation was solved by a MathCad differential equation solver. Satisfactory beam transport was achieved when magnetic field was set to 3 T in the centers of coils with effective length of 67 mm.

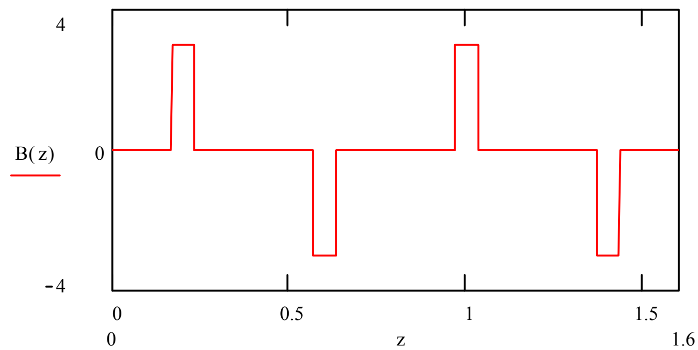

Fig. 2. Magnetic field distribution

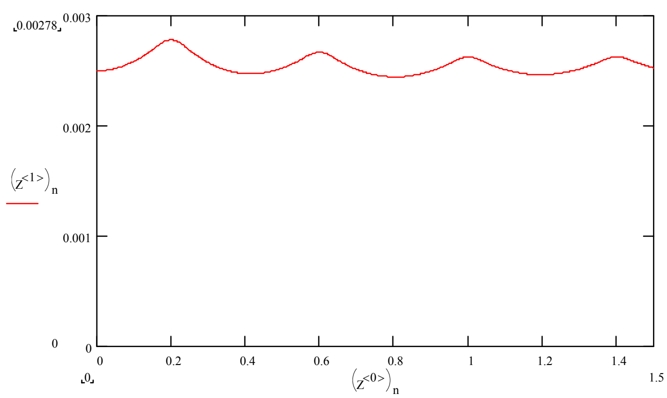

Fig. 3. Beam envelope per Lee-Cooper model.

Change of the initial emittance value requires different setting of magnetic field, change in current also results in a need for field adjustment, but neither of these two parameters seems near of any kind of a threshold, so there is some freedom for field setting.

The envelope equation can use any magnetic field and accelerating electrical field profiles. You just need to invent an analytical representation for these functions. **Smooth magnetic field generated by a thick coil, when used for the equation solving, gave the same envelope if the quantity B^2L was kept constant, which is consistent with what one would expect.**

4 DTL Transport Channel Analysis Using TRACE-3D

After a first order solution to the transport problem was found by using thin focusing solenoids, it was necessary to verify this by using a code that takes into the account longitudinal motion. TRACE-3D, that was used for this purpose, is a widely used code for calculation of the envelope of a bunched beam [3]. It takes into the account beam space charge and longitudinal motion. Because it uses transfer matrix approach, there are some limitations for transport element representation. **For solenoid focusing element, one must use maximal magnetic filed and effective length so that to have a desired value of B^2L.**

Certain details of input that differ from what was discussed earlier are given below. Current input value is the average current through the beam macro-pulse. In our case it is XI = 25 mA. Input emittance values refer to the total the beam current, so, in each transverse it is five times the rms emittance. The values of emittace are not normalized. It means that in the transverse dimension we deal with geometrical characteristics of the beam, not with the motion integral. So, staring from the normalized rms emittance of 0.3 π·mm·mrad, we come to a transverse emittance input value of 19 π·mm·mrad. Longitudinal emittance is fully defined by the energy spread and the phase length: ε_L = 400 π·deg·keV.

The program does not require providing initial radial size of the beam and envelope angle at the input. Instead it requires input of initial elements of Twiss matrix. In our case, we demand that the beam envelope be parallel to the axis at the input and the longitudinal amplitude be maximal, so

$$\alpha_x = \alpha_y = \alpha_\Theta = 0.$$

Beta functions can be found if we assume the beam size as it was done for the envelope equation case:

$$X_m = Y_m = 5.5 \text{ mm}$$

This gives $\beta_x = \frac{X_m^2}{\varepsilon_x} = 1.6\ mm/mrad$, $\beta_y = \frac{Y_m^2}{\varepsilon_y} = 1.6\ mm/mrad$

For the longitudinal motion, $\beta_\Theta = \frac{(\Delta\varphi)^2}{\varepsilon_\Theta} = 1\ deg/kV$

Input file is provided below:

```
ER = 939.30140;      Q= 1.  W = 3.00000;  XI= 25.000;
EMITI = 20.000;      20.000;         400.00;
BEAMI = 0.00;         1.6;    0.00;   1.6;     0;       1
FREQ =   325.000;    PQEXT = 0;
ICHROM = 0;          XC=   0.0000
```

To get uniform envelope, magnetic field in focusing elements was set to smaller values, which are about 25 kGs instead of 30 kGs in the case of the general envelope equation.

Corresponding output file is shown in the picture in Fig. 4.

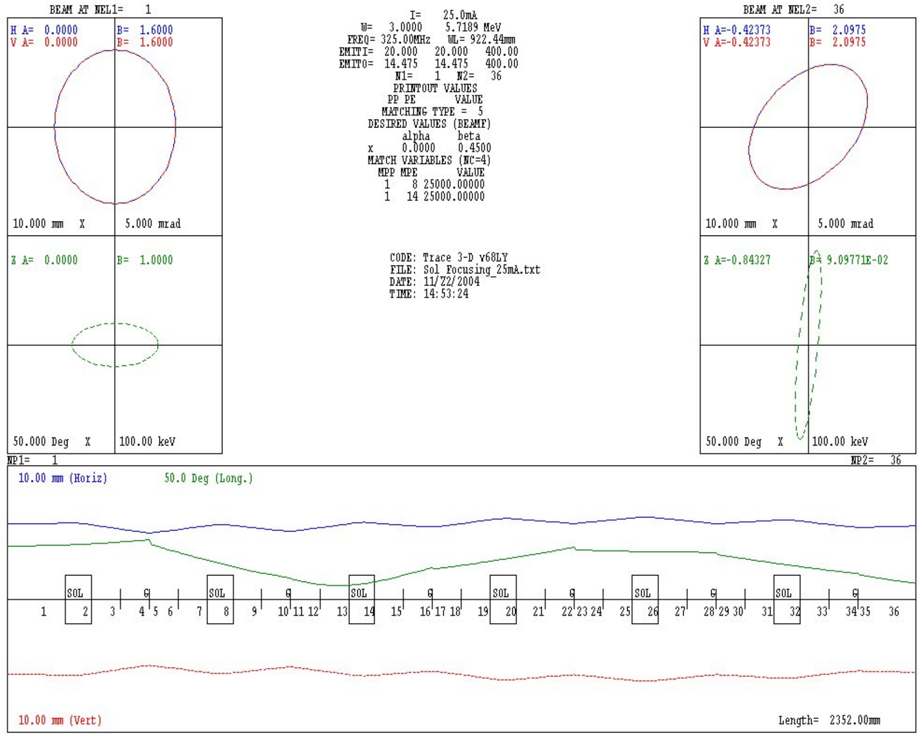

Fig. 4. TRACE-3D output

Comparing traces in Fig 3 and Fig. 4, one can get an impression that at the initial stage TRACE-3D results show less beam expansion that in the case of using the envelope equation (EE).

Because it is important to be sure in the instrument you use to make analysis, I compared the results of calculations of a free expansion zone made by EE and TRACE-3D with the theoretical prediction that consider space charge as a source of a radial force.

Total number of particles in each bunch is defined only by the average current:

$$N = \frac{I_{av}}{q \cdot f}.$$

Knowing the longitudinal dimension of the bunch, one can estimate Coulomb forces and find a trajectory of a particle on the edge of the beam (the envelope). Calculations made for the beam parameters used in previous analysis result in the expected radial relative expansion of 1.06 at the distance of 20 cm from the point of the beam entrance. Both programs gave result close to the predicted by theory, so one can trust the results.

When one needs smaller beam radius, higher magnetic field is required, and maybe less spacing between the coils. Runs made for R_{rms} = 0.9 mm (R_{nax} = 2.1 mm) show that it is still possible to confine the beam within acceptable radial limits. In accordance with the EE model, the first lens, located 20 mm from the input aperture, must have 5T field in its center. TRACE-3D requires 4T focusing fields. The difference is due to different scaling of the space charge forces with radius for the two models. For a long beam, the force scales as 1/R, and for short bunches it scales as $1/R^2$, so less focusing strength is required to confine the beam within needed aperture. Corresponding traces are compared in Fig. 7 and 8.

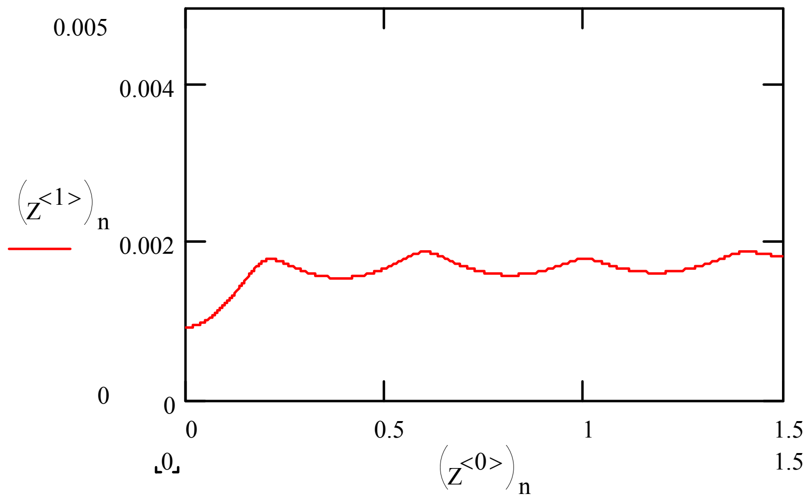

Fig. 7 Transport of the beam with initial radius of 2.1 mm – EE.

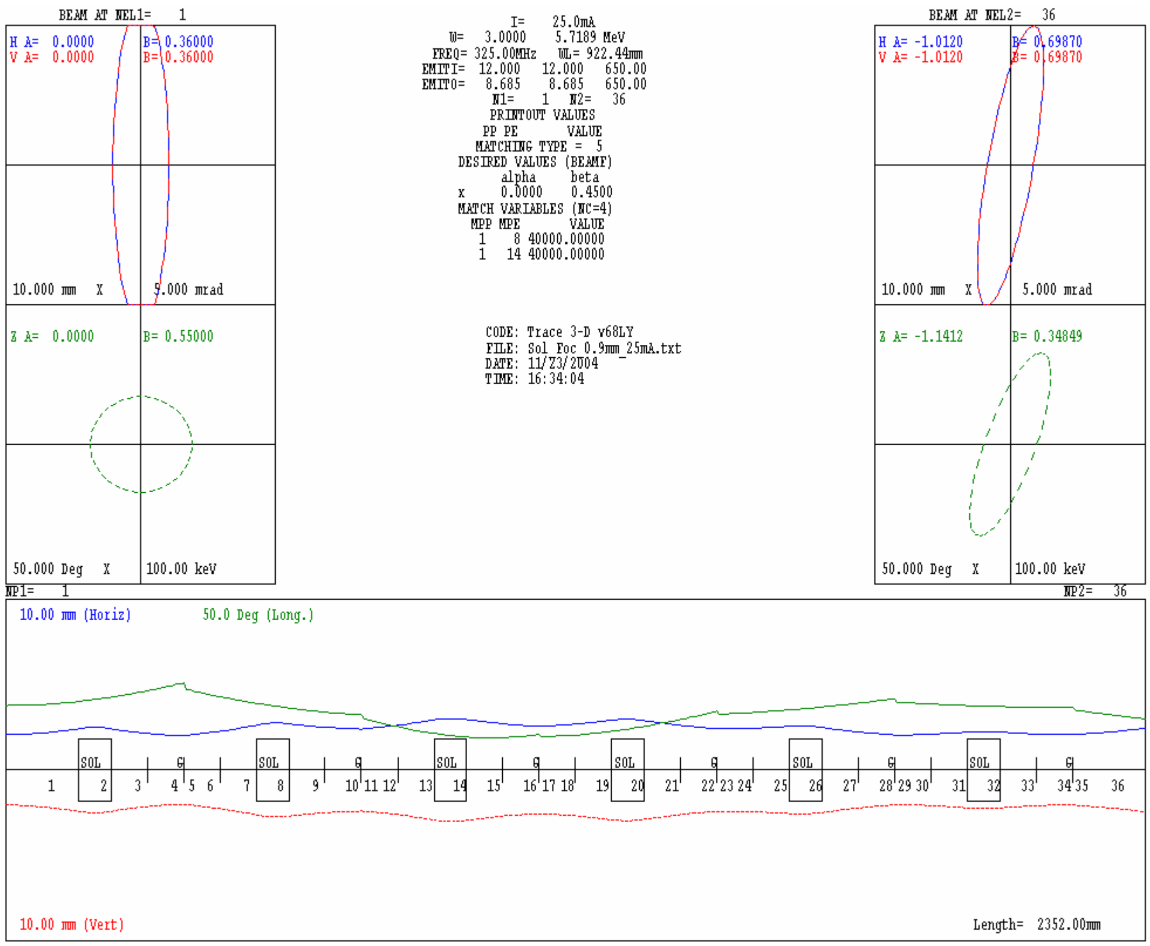

Fig. 8 Transport of the beam with initial radius of 2.1 mm – TRACE-3D.

5. Summary

As a mean to avoid excessive power loss due to halo effects in a beam of a Proton Driver, focusing using short solenoid channel is considered for a part of a PD front end. Short study of the issue has shown that there is no immediate "show stoppers" and that it is possible to further develop this approach, although a lot of work is ahead in the fields of the accelerating system development and beam tracking in order to chose right solenoid parameter range and design approach.

6. References

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